Int. J. Multiphase Flow, Vol. 2, pp. 213–218. Pergamon/Elsevier, 1975. Printed in Great Britain

# BRIEF COMMUNICATION

# DEPOSITION OF PARTICLES FROM A GAS FLOWING PARALLEL TO A SURFACE

G. C. GARDNER

Central Electricity Research Laboratories, Leatherhead, Surrey, England

(Received 20 July 1974)

## INTRODUCTION

The practising engineer or designer frequently requires an estimate of the rate of deposition of particles from a turbulent gas stream flowing parallel to a surface. The mechanism for deposition changes with particle size, as a glance at the mass transfer coefficients in figure 1 indicates, and the the descriptions for each mechanism, which are widely distributed in the literature, are often couched in terms awkward for direct application. The purpose of this paper is therefore to present means for the calculating deposition rate in an amenable form.

Figure 1 shows that the smallest particles are deposited by a diffusion process. However, a particle size is reached, above which the inertia of the particles becomes such that the velocity given to them by turbulent eddies is not dissipated rapidly in the boundary layer. Then, as Friedlander & Johnstone (1957) first showed, the deposition rate rises abruptly in what is termed the eddy diffusion–impaction regime. Nonetheless the mass transfer rate cannot rise above a value of the order of magnitude of the friction velocity, since that approximately equals the average turbulent eddy velocity towards the wall. In fact, Liu & Agrawal (1974) showed that the transfer coefficient changes fairly abruptly to a slowly falling value with respect to particle size. As the particle size rises still further, it is found that the particle inertia becomes so large that particles cannot attain the eddy velocity during the time they are caught up by an eddy and, in consequence, the deposition rate falls more and more rapidly. The theory of Hutchinson *et al.* (1971) is applicable in this region.

Superimposed on all the above effects for electrically neutral particles are other influences which can be reduced to consideration of equilibrium velocities normal to the surface. Figure 1 shows the influence of some thermopheretic forces which give an equilibrium velocity away from

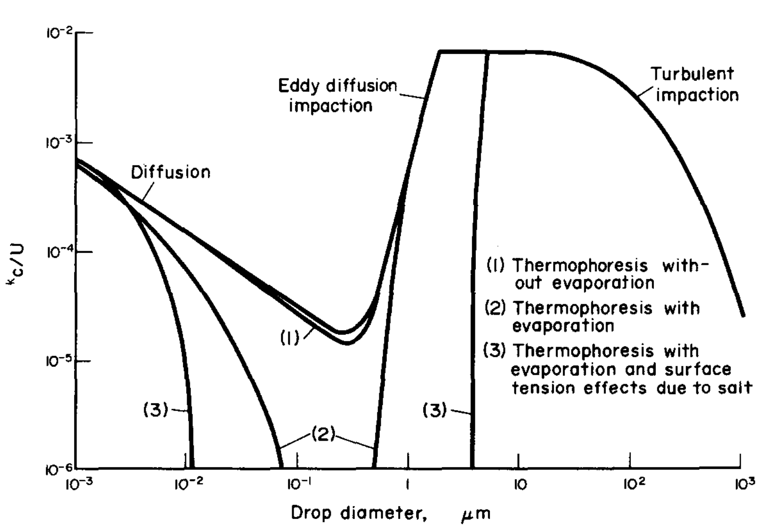


Figure 1. Deposition of drops from 180 bar steam being heated in a 1.3 cm diameter tube. Heat flux 0.3 MW/m$^2$; steam velocity 12.3 m/sec.

the surface but another obvious force is due to gravity, which was considered by Sehmel (1973). The reader may like to note that Epstein (1929), Waldmann (1959) and Brock (1962) have dealt with thermophoresis, Waldmann (1959), Brock (1963) and Derjaguin *et al.* (1966) have dealt with diffusiophoresis, Gardner (1968) has dealt with thermophoresis of an evaporating particle, Yalamov & Shchukin (1971) have dealt with thermo-diffusiophoresis and Gardner (1975) has considered thermophoresis of an evaporating drop with surface tension gradients caused by concentration gradients of a solute.

The methods of calculation will be presented in two parts. The first concerns the diffusion and eddy diffusion–impaction regimes with a superimposed equilibrium velocity, which usually has major impact within those regimes. The second concerns the turbulent impaction regime shown in figure 1.

## THE DIFFUSION AND EDDY DIFFUSION-IMPACTION REGIMES

Consider a particle concentration $c$ in the boundary layer. The mass transfer rate $N$ is given by

$$N = (D + D_\epsilon)\frac{\mathrm{d}c}{\mathrm{d}y} + u_E c, \qquad [1]$$

$D$ is particle diffusivity which is usually given by Einstein's equation

$$D = \frac{kT}{3\pi\mu d}, \qquad [2]$$

where $D_\epsilon$ is the eddy diffusivity, $y$ is distance from the surface, $u_E$ is the equilibrium velocity of the particle towards the surface, which is obtained by a balance of viscous resistance to motion according to Stokes law and, say, a thermophoretic or gravitational force, $k$ is Boltzmans constant, $T$ is absolute temperature, $\mu$ is the gas velocity and $d$ is the particle diameter.

Now the principal changes in concentration with material of low diffusivity, such as the particles of concern, occurs in the laminar sublayer when Lin *et al.* (1953) showed that

$$D_\epsilon = 3{\cdot}28\ 10^{-4}\nu y^{+3}, \qquad [3]$$

where $\nu$ is the kinematic viscosity of the gas,

$$y^+ = \frac{u^* y}{\nu}, \qquad [4]$$

and $u^*$ is the friction velocity

$$u^* = \left(\frac{f}{2}\right)^{1/2} U, \qquad [5]$$

where $f$ is the friction factor and $U$ is the mainstream velocity parallel to the surface.

Equation [1] can now be integrated from $y^+ = \infty$, where $c = c_\infty$, to a value of $y^+ = y_1^+$ at which the concentration is sufficiently small for it to be set equal to zero, without introducing substantial error. It is noted that, strictly speaking, the concentration is only zero at $y^+ = 0$. The following equation is thus obtained for the mass transfer coefficient $k_c = N/c_\infty$.

$$\frac{k_c}{u^*} = \frac{(u_E/u^*)}{1 - E/F}, \qquad [6]$$

where

$$E = \exp\left[\frac{B\nu u_E}{3^{1/2}Du^*}\left(\tan^{-1}\left(\frac{2y_1^+ - B}{3^{1/2}B}\right) - \frac{\pi}{2}\right)\right], \qquad [7]$$

$$F = \left[\frac{(B + y_1^{+2})^2}{B^2 - By_1^+ + y_1^{+2}}\right]^{[-(B\nu u_E/6Du^*)]}, \qquad [8]$$

$$B = 14.5\left[\frac{D}{\nu}\right]^{1/3}. \qquad [9]$$

It is now assumed that particles have a velocity $(Au^* + u_E)$ towards the surface at $y^+ = y_1^+$ and that this velocity decays to zero at $y^+ = 0$ due to a Stokes law resistance. The assumption that part of the initial velocity is proportional to the friction velocity has been indicated by Owen (1969) to be compatable with modern concepts of turbulent bursts in the boundary layer.

The equation of motion is

$$\frac{\pi}{6}d^3\rho_P\frac{du}{dt} = -3\pi\, d\mu(u - u_E), \qquad [10]$$

where $u$ is velocity, $t$ is time, $\rho_P$ is particle density and $\mu$ is gas viscosity.

Double integration of [10] yields

$$y_1^+ = \frac{A}{18}\left(\frac{\rho_P}{\rho}\right)\left(\frac{du^*}{\nu}\right)^2\left[1 + \frac{u_E}{Au^*}\right]. \qquad [11]$$

Equation [6] is now readily evaluated with a desk calculator and examples of the result are given in figure 2. $A$ has been chosen equal to 0.957 in agreement with the accurate results of Liu & Agrawal (1974) for the eddy diffusion–impaction regime. This regime and that for diffusion are obvious in figure 2. The influence of the equilibrium velocity is also clear.

It must be emphasised that the separate mechanisms for deposition inherent in [6] have been tested. With respect to the eddy diffusion–impaction regime, a more complex expression for the eddy diffusivity than that of [3] has usually been chosen but this equation is found adequate in representing the experimental results and allows a relatively simple set of equations, marrying all the mechanisms, to be formulated. The place where more experimental work is required is with respect to the interaction of an equilibrium velocity with the other mechanisms but it is noted that deposition rate often changes so rapidly with particle size that the designers inaccurate knowledge of that size will usually be the chief factor in the assessments he

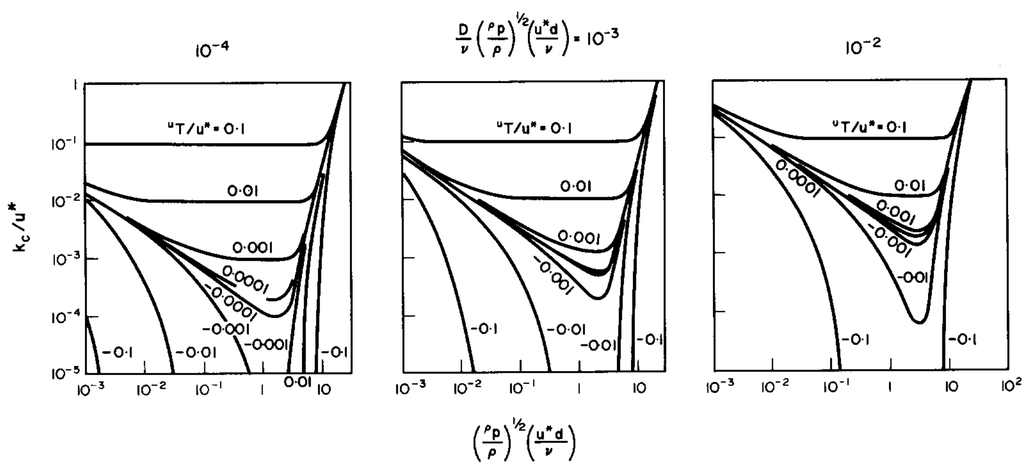


Figure 2. Mass transfer coefficient for particle deposition.

makes. He will then wish to know when a particular equilibrium velocity, say due to heat transfer, will be dominant in either preventing any substantial deposition or in controlling the deposition rate.

## THEORY FOR TURBULENT IMPACTION

The turbulent impaction theory of Hutchinson *et al.* (1971) is complex in form and the following aims at a convenient reduction of the results. The theory is in two parts. The first part was presented as an efficiency $\eta$ of particle deposition in terms of $(x/d_T)$, where $x$ is tube length and $d_T$ is tube diameter, and a parameter $\gamma$

$$\gamma = 0{\cdot}11\,\overline{h^2}\left(\frac{f}{2}\right)^{1/2}, \qquad [12]$$

$h$ is the distance travelled by a particle in a single eddy divided by the eddy size. However, the theory shows that the efficiency is solely dependent upon $(\gamma x/d_T)$ in the fashion shown by figure 3. This figure indicates that, after an entry length, $\ln(1-\eta)$ is linear in $(\gamma x/d_T)$ and this is readily interpreted in terms of a constant mass transfer coefficient

$$\frac{k_c}{U} = 1{\cdot}445\gamma. \qquad [13]$$

Hutchinson in a private communication has indicated that the numerical coefficient in [13] can be determined from the first term in an expansion of [20] in Hutchinson *et al.* (1971) paper.

Equations [12] and [13] yield

$$\frac{k_c}{u*} = 0.159\,\overline{h^2}. \qquad [14]$$

The second part of the theory was presented as a series of graphs of $\overline{h_R^2} = 0.5\overline{h^2}$ vs the ratio $(\rho/\rho_p)$. Each graph was for a specific value of $(d/d_T)$ and contained a family of curves, with the pipe Reynolds number as a parameter. $(d/d_T)$ had the range $10^{-4}$–$10^{-2}$, $(\rho/\rho_p)$ had the range $2\,10^{-4}$–$5\,10^{-3}$ and the Reynolds number the range $10^4$–$5\,10^5$. It is noted that some extrapolation is necessary for some problems, especially with respect to the density ratio.

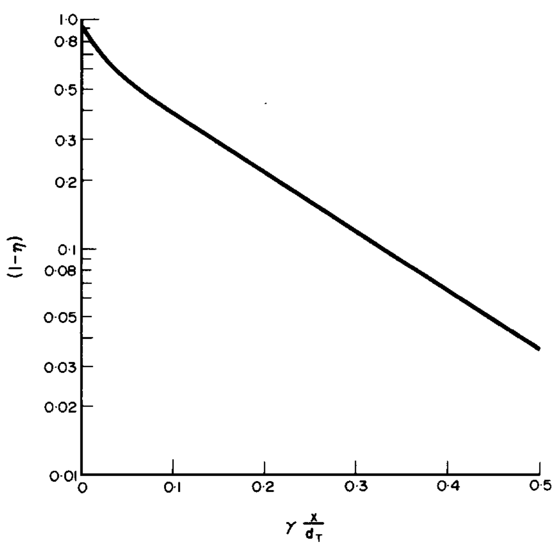


Figure 3. Hutchinson, Hewitt & Dukler's theory for the impaction collection of large particles.

In order to formulate a simple empirical correlation of $\overline{h^2}$ it was assumed that a particle of initially zero velocity was caught up by an eddy with velocity $u^*$. The eddy was assumed to have a lifetime proportional to $(d_T/u^*)$ and in consequence $h$ is found to be proportional to $[(\rho/\rho_p)(d_T/d)C_D]$, where $C_D$, the drag coefficient, is a function of

$$(Re)_p = \frac{Ud}{\nu}. \qquad [15]$$

It was found that a good approximation is that

$$\overline{h^2} = H\left[\frac{\rho}{\rho_p}\frac{d_T}{d}(Re)_p{}^{(-0.72+0.023\ln \overline{h^2})}\right], \qquad [16]$$

where $H$ represents "function of". This conclusion is illustrated in figure 4. Also shown in the figure are the experimental results from Liu & Agrawal (1974) outside the eddy diffusion–impaction regime and it is seen that their results indicate a limiting value of $\overline{h^2}$ of about 0.88 rather than 5.1 for Hutchinson *et al.* (1971) theory. This may seem to cast doubts on the latter but it must be emphasised that the theory was extensively checked against experiment for large particles and drops away from limiting value of $\overline{h^2}$ and it should be valid in that region. However, until further experiments are available to compliment the apparently accurate work of Liu & Agrawal, it is unwise to expect values of $k_c/u^*$ greater than 0.14 in correspondence with $\overline{h^2} = 0.88$.

It should be noted that Hutchinson *et al.* extended their theory to consider interactions of particles with the boundary layer and tested it against the experimental results of Friedlander & Johnstone (1957), amongst others, for the eddy diffusion–impaction regime. They found good agreement after selecting an adjustable parameter. However, the simple and well proven analysis given in the last section is more satisfactory for general use.

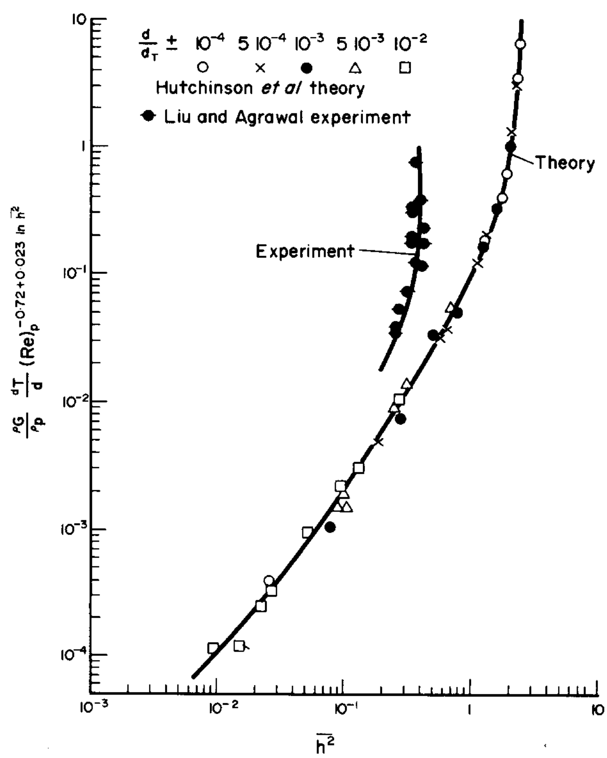


Figure 4. Correlation of $\overline{h^2}$.

Table 1

| $H$ | $\overline{h^2}$ Hutchinson |
|---|---|
| 5 | 4.8 |
| 2 | 4.5 |
| 1 | 4.1 |
| $5\ 10^{-1}$ | 3.72 |
| $2\ 10^{-1}$ | 2.66 |
| $1\ 10^{-1}$ | 2.04 |
| $5\ 10^{-2}$ | 1.44 |
| $2\ 10^{-2}$ | 0·84 |
| $1\ 10^{-2}$ | 0.55 |
| $5\ 10^{-3}$ | 0.36 |
| $2\ 10^{-3}$ | 0.194 |
| $1\ 10^{-3}$ | 0.120 |
| $5\ 10^{-4}$ | 0.072 |
| $2\ 10^{-4}$ | 0.0346 |
| $1\ 10^{-4}$ | 0.0190 |

The coordinates of the theoretical curve of figure 4 are given in the above table.

Lastly it is noted that the entry length illustrated in figure 3 and required for the mass transfer coefficient to settle down to its constant value is about 15 pipe diameters, if $\overline{h^2}$ is 0.88.

*Acknowledgement*—This work was carried out at the Central Electricity Research Laboratories and is presented by permission of the Central Electricity Generating Board.

REFERENCES

BROCK, J. R. 1962 On the theory of thermal forces acting on aerosol particles. *J. Colloid Sci.* **17**, 768–780.

BROCK, J. R. 1963 Forces on aerosols in gas mixtures. *J. Colloid Sci.* **18**, 489–501.

DAVIES, C. N. 1966 *Aerosol Science* (Edited by DAVIES, C. N.). Academic Press.

DERJAGUIN, B. V., YALAMOV, YU. I. & STOROZHILOVA, A. I. 1966 Diffusiophoresis of large aerosol particles. *J. Colloid Interface Sci.* **22**, 117–125.

EPSTEIN, P. S. 1929 Zur Theorie des Radiometers. *Z. Phys.* **54**, 437–563.

FRIEDLANDER, S. K. & JOHNSTONE, H. F. 1957 Deposition of suspended particles from turbulent gas streams. *Ind. Engng Chem.* **49**, 1151–1156.

GARDNER, G. C. 1968 Force on a small spherical evaporating particle in its pure evapour due to motion and temperature gradients. *Chem. Engng Sci.* **23**, 29–40.

GARDNER, G. C. 1975 Surface tension gradients and the motion of an evaporating drop containing salt in a temperature gradient. *Aerosol Sci.* **6**, 173–179.

HUTCHINSON, P., HEWITT, G. F. & DUKLER, A. E. 1971 Deposition of liquid or solid dispersions from turbulent gas streams; a stochastic model. *Chem. Engng Sci.* **26**, 419–439.

LIN, C. S., MOULTON, R. W. & PUTNAM, G. L. 1953 Mass transfer between a solid wall and fluid streams. *Ind. Engng Chem.* **45**, 636–646.

LUI, B. Y. H. & AGRAWAL, J. K. 1974 Experimental observation of aerosol deposition in turbulent flow. *Aerosol Sci.* **5**, 145–155.

OWEN, P. R. 1969 Pneumatic transport. *J. Fluid Mech.* **39**, 407–432.

SEHMEL, G. A. 1973 Particle eddy diffusivities and deposition velocities for isothermal flow and smooth surfaces. *Aerosol Sci.* **4**, 125–138.

WALDMANN, L. 1959 Über die Kraft eines inhomogenen Gases auf kleine suspendierte Kugeln. *Z. Naturforsch.* **149**, 589–599.

YALAMOV, YU. I. & SHCHUKIN, E. R. 1971 Thermophoresis of evaporating drops in coarse aerosols. *Russian J. Phys. Chem.* **45**, 1375–1377.